$\mathbb{Z}_2$ Spin Liquids and the Mott Insulator*

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Spin liquids are phases are known for their lack of long range magnetic order and fractionalized quasiparticles. While fractionalization has been understood in 1D antiferromagnet at $S = 1/2$ via the Bethe ansatz [1], extending this solitonic mechanism to higher dimensions had been difficult. Other mechanisms of fractionalization were then sought out for, with 2d spin-charge separation realized in considering fluctuations over a mean field theory approach from a quantum spin model or the t-J model [2–4] . However, it was unclear whether this fractionalization would survive the resulting strongly interacting gauge theory model. In their work [5] , Senthil and Fisher demonstrate a general framework to study fractionalization in strongly correlated systems which can be extended to arbitrary spatial dimension and spin-rotation non invariant systems. They studied a specific class of microscopic models which interpolate between an antiferromagnetic Mott Insulator and a conventional d-wave superconductor by invoking $\mathbb{Z}_2$ gauge theory. $\mathbb{Z}_2$ vortices are realized to play a vital role in both the emergence of fractionalization and the appearance of a Mott Insulating state. Since the Mott Insulator is a phenomen born out of strongly correlated interactions, its realization is a sign that their framework accurately describes strong electron-electron interactions. In this paper, we will focus on that state, a type of Mott Insulator called the nodal spin liquid [6].

I. MICROSCOPIC MODEL

We first introduce the model studied in [5]. Consider a generalized Hubbard model on a 2d square lattice with local d-wave pairing fluctuations, $\mathcal{H} = \mathcal{H}_0 + \mathcal{H}_J + \mathcal{H}_u + \mathcal{H}_\Delta$,

$$H_0 = -t \sum_{\langle rr'\rangle} c^\dagger_{r\alpha} c_{r'\alpha} + h.c. \qquad \text{(Hopping)}$$

$$\mathcal{H}_J = J \sum_{\langle rr'\rangle} \mathbf{S}_r \cdot \mathbf{S}_{r'} \qquad \text{(Heisenberg)}$$

$$\mathcal{H}_u = \sum_r u(N_r - N_0)^2 \qquad \text{(On-site)}$$

$$\mathcal{H}_\Delta = \Delta \sum_r \left(e^{\mathrm{i}\varphi_r} p_r + h.c.\right) \qquad \text{(Pairing)}$$

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with local d-wave pairing field,

$$p_r = \sum_{r'\in r} \Delta_{rr'}(c_{r\uparrow}c_{r'\downarrow} - c_{r\downarrow}c_{r'\uparrow}) \tag{1}$$

and $\Delta_{rr'} = \Delta$ for bonds along the x-direction and $\Delta_{rr'} = -\Delta$ for bonds along the y-direction so that p_r destroys a $d_{x^2-y^2}$ pair of electrons centered at the site r. In this model, $c_{r\alpha}$ is the annihilation operator of an electron with spin α at site r. We may split this electron by introducing a chargon operator b_r and spinon operator $f_{r\alpha}$ with,

$$c^\dagger_{r\alpha} = b^\dagger_r f^\dagger_{ra} \tag{2}$$

where $b_{r\alpha}$ is the chargon annihilation operator and $f_{r\alpha}$ is the spinon annihilation operator. The chargon is a spinless chargeful boson while the spinon is a $s = 1/2$, chargeless fermion.

After this electron substitution, projecting back to the physical Hilbert space using a $\mathbb{Z}_2$ gauge field, and retaining a particular set of fluctuations about the saddlepoint of a Hubbard Stratanovich decoupling of the spin interaction term [7], Senthil and Fisher arrive to a chargon spinon coupled to a $\mathbb{Z}_2$ gauge field σ_{ij} living on the links of the lattice site $S = S_c + S_s + S_B$.

$$S_c = -2t_c \sum_{\langle ij\rangle} \sigma_{ij} \cos(\phi_i - \phi_j), \tag{3}$$

$$S_s = -\sum_{\langle ij\rangle} \sigma_{ij} \left(t^s_{ij} \bar{f}_i f_j + t^\Delta_{ij} f_{i\uparrow} f_{j\downarrow} + c.c.\right) - \sum_i \bar{f}_i f_i \tag{4}$$

where $\exp(-S_B) = 1$ for an even density of electrons. In this case, the system realizes a band insulating state for $t_c \ll 1$ and a BCS superconducting state for $t_c \gg 1$. While this even density case also realizes a fractionalized insulat state due to a confinement-deconfinement transition of spinons/chargons, we will focus more on the case of an odd density of electrons for which the Mott Insulator phase appears.

II. MOTT INSULATOR: ODD ELECTRON DENSITY

Mott Insulators are insulators that under standard band theory would be classified as conductors, but due to electron-electron interactions, they act as insulators. In the simplest case, we consider one electron per unit cell so that the conduction band is half-filled [8].

Under the class of microscopic models described above, exploring the odd density of electrons means considering a Mott Insulator.

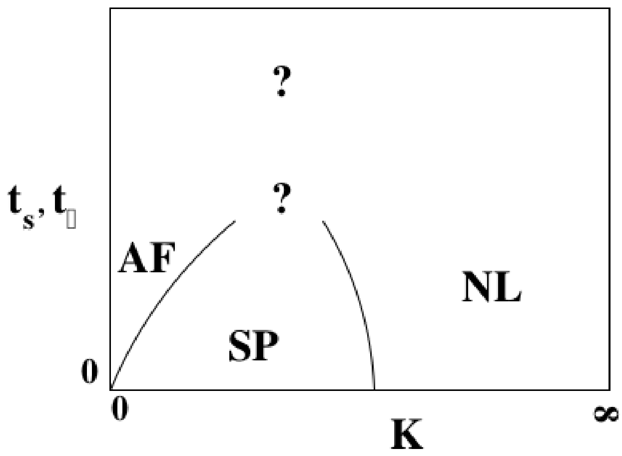

FIG. 1: Phase diagram $t_c, t_\triangle$ vs. K for microscopic model with odd electron density and $t_c \ll 1$. As one increases K for low $t_s, t_\triangle \neq 0$, the antiferromagnetic insulator (AF) evolves to a conventional spin-Peierls state (SP) to a fractionalized nodal liquid (NL).

Then, the Berry action is non-trivial,

$$S_B = -\mathrm{i}\frac{\pi}{2} \sum_{i,j=i-\hat{\tau}} (1 - \sigma_{ij}) \tag{5}$$

If $t_c \ll 1$, the chargons cannot propogate and will be gapped out. We can integrate out the chargons from S_c and be left with a plaquette product term in its stead, $S = S_\sigma + S_s + S_B$,

$$S_\sigma = -K \sum_{\square} \left[\prod_{\square} \sigma_{ij} \right] \tag{6}$$

If $K = 0$, this action formally reduces to the Heisenberg antiferromagnetic spin model,

$$\mathcal{H} = J \sum_{\langle rr' \rangle} \mathbf{S}_r \cdot \mathbf{S}_{r'} \tag{7}$$

and we attain an antiferromagnetic insulator. If $K = \infty$, the gauge fields are effectively frozen out and we can choose $\sigma_{ij} = 1$ for every spatial link. Then, the spinons propogate under t^s_{ij} spinon coupling and $t^\triangle_{ij}$ d-wave spinon pairing. This d-wave pairing induces a gapless "d-wave" dispersion at four nodes in the Brillouin zone. This is therefore a fractionalized insulator with deconfined, gapless spinons, gapped chargons, and gapped visons: a nodal spin liquid. This nodal spin liquid remains stable for finite K but across a critical coupling K_c undergoes a confinement transition to a translational symmetry breaking order: the spin-Peierls state. While it is difficult to calculate a precise phase diagram, considering different limits of $t_s, t_\triangle$, and K indicate a phase diagram of the form shown in Figure 1.

If $t_c \gg 1$, we enter the superconducting phase. In this case, the chargons condense so that $\langle e^{\mathrm{i}\varphi} \rangle \neq 0$. This condensation breaks both the $U(1)$ charge symmetry and $\mathbb{Z}_2$ gauge symmetry. However, one may show that the $U(1)$ hc/e vorticies are confined unless they are bound to the $\mathbb{Z}_2$ vortex. In fact, since the chargons are a relative semion with respect to both the U(1) vortex and the $\mathbb{Z}_2$ vortex, the U(1) vortex will acquire a phase $hc/2e$. This way, the chargon acquires a phase of π revolving around the U(1) vortex and $\mathbb{Z}_2$ vortex pair. Moreover, since the spinon is a relative semion with respect to the $\mathbb{Z}_2$ vortex, it "sees" the vortex pair [9]. The appearance of massive hc/2e vortices [10] and spinons implies that we have recovered the d-wave superconductivity.

In fact, the nodal spin liquid can be further understood as a descendant of the BCS superconductivity. Using boson-vortex duality [11–15] , we can trade the chargon fields for its respective $hc/2e$ vortices such that the superconducting phase is represented by the vortex vacuum while the insulating phase is the viewed as the vortex condensate. If one starts in the d-wave superconducting state, pairs the BCS hc/2e vortices, then condenses the resultant hc/e vortices, the resulting action describes gapped visons, gapped chargons, and gapless spinons, i.e. the nodal liquid. This perspective demonstrates that the vison is the fragment of the BCS hc/2e vortex after they condensed into pairs [5].

While this microscopic description may come out of controling the tuning parameters, this transition from BCS superconductor to nodal spin liquid can occur by introducing quantum disorder to the superconductor, i.e. allowing the phase field of the order parameter to fluctuate greatly [9]. This analysis demonstrates the spinon of the nodal liquid (or nodon) is a remnant of the low energy, gapless quasiparticles of the d-wave superconductor.

This perspective implies that the nodal spin liquid is a descendant of the d-wave superconductor.

III. OUTLOOK

Many aspects of Senthil and Fisher's paper were not new. The fractionalization mechanism discussed here can actually be shown to be equivalent to the vortex-pairing mechanism developed by Balents. [16, 17] Even using $\mathbb{Z}_2$ Gauge theory to describe fractionalization was not new. Previous works showed that the Sp(2N) antiferromagnet at large N with frustration [18, 19] or in quantum dimer models with frustration [20, 21] reduce to a $\mathbb{Z}_2$ Gauge theory. X.G. Wen proposed pairing and condensing pairs of spinons in an conventional SU(2) Heisenberg magnet by to reduce the gauge symmetry to $\mathbb{Z}_2$. So what was the innovation? Senthil and Fisher's innovation lay in their ability to generalize this vortex pairing description via $\mathbb{Z}_2$ gauge theory to a large class of models for arbitrary dimension while capturing strongly correlated physics via the realization of the nodal spin liquid.. While it is not clear if these models accurately describe cuprate superconductors or other strongly correlated materials, this paper seemed to be essential in the development of modeling fractionalized exotic phases.

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